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A Scalar Field Theory of Gravitation

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Abstract

A scalar theory of gravitation is developed from a variational principle. The speed of light is taken to be a function of the potential of the gravitational field. The predictions of the light deflection and the advancement of the perihelion agree with those made by Einstein's theory. The gravitational (active) mass differs from the inertial (passive) mass and both are dependent on the gravitational potential.

1. *Introduction*

The most serious objection raised against scalar theories of gravitation has been the inability to correctly predict the light deflection. Indeed, in the class of generalised Nordström theories considered by Whitrow & Murdoch (1965) it is shown that the perihelion advancement may be reproduced but not the light deflection. In addition to these Lorentz covariant theories, Whitrow and Murdoch also considered theories with a variable speed of light although their treatment was not consistent from the point of view of either tensor calculus or field theory. The idea of a variable speed of light is not new, Einstein (1911) had considered it. Rosen (1940) considered a flat-space interpretation in which the speed of light was dependent on the 'scalar' component of the gravitational field. If one accepts the hypothesis of the variability of the speed of light, in particular that it depends on the gravitational field, then it is consistent with electromagnetic theory to consider the light deflection simply as the refraction through a medium with the appropriate index of refraction. It is a simple matter to show that the speed of light must be proportional to 1 minus twice the magnitude of the Newtonian potential in order that an application of Fermat's principle should give a result agreeing with the prediction of general relativity. The testability of this hypothesis is on the boundaries of the limits of present technology. An earth-based laboratory would record a variation in the speed of light by 2/3 parts in 10^9 in the course of the annual motion.

According to Froome & Essen (1965) one can measure the speed of light to an accuracy of 1 part in 10^8 with present-day technology. A satellite in a highly eccentric solar orbit would deliver far better results. One of the first scalar theories of gravitation was developed by Nordström (1912). This theory and the theory of Bergmann (1956) have been discussed in a previous article (Lindén, 1971). An historical account of this very active period in the development of gravitational theories has been given by Guth (1970). The principal features of Nordström's first theory are: (1) global Lorentz invariance (constant speed of light), (2) the field satisfies a wave equation and (3) the four-force acting on a body is proportional to the four-gradient of the potential and Newton's law of motion is generalised to a four-dimensional description. The second theory of Nordström (1913) is also globally Lorentz invariant, however, the field equation is altered in that the D'Alembertian operator is multiplied (from the left) by the field and the four-force is now given by the product of the trace of the energy-momentum tensor of matter and the four gradient of the logarithm of the field. Abraham (1914) considered two locally Lorentz invariant theories of gravitation. In the first theory (Abraham, 1912a)—which preceded Nordström's first theory—the equations of motion and the field equation are the same as the first Nordström theory except the speed of light is not constant but is related to the field by a formula similar to the one hypothesised by Einstein (1911). In the second theory of Abraham (1912b), which also preceded the first theory of Nordström, the field quantity is the square root of the speed of light. This field quantity satisfies a Klein–Gordon type equation with the 'field-stiffness' (the mass term in the Klein–Gordon equation) given by the rest mass density of matter. The four-force is the gradient of the speed of light. In both of these theories, the light deflection is only half of the amount predicted by general relativity.

In the present paper, a scalar theory of gravitation is developed which is related to the acknowledged papers in content and to the paper of Bergmann (1956) in method. Thirring (1961) also pursued the field-theoretic approach although he emphasised a two-tensor theory.

2. *The Field Equations*

The gravitational field is taken to be described by a scalar potential. It is further hypothesised that the speed of light depends on the field. The nature of this dependence is for the time being left unspecified. We should like to formulate a variational principle from which the equations of motion and the field equations follow by performing the appropriate variations. We thus require a Lagrangian density. This Lagrangian must consist of three terms, the free field term, the free particle term and the interaction term. The free particle term and the free field term we know how to construct if we prescribe how the equations of motion are to be parametrised (i.e. whether the equations of motion should follow from the variation of $\int ds$ or $\int d\tau$ where s is the world distance and τ the proper time). I see no *a priori* reason for choosing one or the other, apart from the fact that intuition prefers τ. We shall, however, choose s as the fundamental parameter. In Bergmann's (1956) theory it makes no difference since his action is invariant under the (twice-differentiable) class of transformations $\lambda \to \lambda(s)$. The Lagrangian we shall choose (as Thirring's (1961)) is only invariant under the one parameter group of translations $s \to s + a$. There remains only the interaction term. In a previous paper, Lindén (1971), an argument was presented to the effect that the interaction term should be a product of the mass density and an expression quadratic in the potential. It is through the linear term that sources of the field are provided. The linear term also provides the mechanism for dependence of inertial (passive) mass upon the field. The quadratic term provides a mechanism for the dependence of gravitational (active) mass upon the field. We are thus in a position to state the Lagrangian density.

$$\mathscr{L} = \tfrac{1}{2}\sqrt{(-g)}\, g^{ij}\, \Phi_i\, \Phi_j + 4\pi\alpha \int ds g_{ij}\, \dot{z}^i\, \dot{z}^j\, \rho(x^i, z^i)(1 + \beta\Phi(x^i))^2 \quad (2.1)$$

The constant α and the coupling constant β will be determined by comparison with Newton's equations of motion.

In equation (2.1) the metric tensor is a function of Φ. In a Cartesian coordinate system we have in particular that

$$g_{ij} = \mathrm{diag}\,(c^2, -1, -1, -1) \quad (2.2)$$

where c is a function of Φ to be described shortly. A dot over a symbol denotes differentiation with respect to s and $\Phi_j = \partial_j \Phi$. The quantity ρ is the density distribution of matter times $\sqrt{-g}$, so that for a point source ρ would simply be the product of four delta functions. The action functional is defined by

$$\mathscr{A} = \int_{\Omega} d^4 x \mathscr{L} \quad (2.3)$$

If this action is to be stationary, then the variation

$$\Phi \to \Phi + \delta\Phi \quad (2.4)$$

where $\delta\Phi$ is an arbitrary function vanishing on the hypersurface $\partial\Omega$, leads to the Euler–Lagrange equation

$$d_i(\sqrt{(-g)}\, g^{ij}\, \Phi_j) - \frac{1}{2}\frac{\partial\sqrt{(-g)}\, g^{ij}}{\partial\Phi}\, \Phi_i\, \Phi_j = 4\pi\alpha \int ds \rho(x^i, z^i)\, \dot{z}^i\, \dot{z}^j \frac{\partial}{\partial\Phi}\, [(1 + \beta\Phi)^2\, g_{ij}] \quad (2.5)$$

where

$$d_i = \frac{d}{dx^i} = \frac{\partial}{\partial x^i} + \Phi_i \frac{\partial}{\partial\Phi}$$

Thus equation (2.5) may be written

$$\partial_i(\sqrt{(-g)}\,g^{ij}\Phi_j) + \frac{1}{2}\frac{\partial\sqrt{(-g)}\,g^{ij}}{\partial\Phi}\Phi_i\Phi_j$$
$$= 4\pi\alpha\int ds\rho(x^i, z^i)\,\dot{z}^i\dot{z}^j\frac{\partial}{\partial\Phi}[(1+\beta\Phi)^2 g_{ij}] \qquad (2.6)$$

Now, if (2.6) is divided by $\sqrt{-g}$ then the first term is the D'Alembert operator acting Φ. The second term is the energy density of the field. We conclude this section with the presentation of (2.6) in special coordinate systems, viz. those in which the metric tensor takes the form

$$g_{ij} = \operatorname{diag}(c^2, -h_1{}^2, -h_2{}^2, -h_3{}^2) \qquad (2.7)$$

where the quantities h_i are known as scale factors. Equation (2.6) then becomes

$$\frac{1}{c}\frac{\partial}{\partial t}\left(\frac{Jc^{1/2}}{c}\frac{\partial\Phi}{\partial t}\right) - \sum_{i=1}^{3}\frac{\partial}{\partial x^i}\left(\frac{Jc^{1/2}}{h_i{}^2}\frac{\partial\Phi}{\partial x^i}\right)$$
$$= 8\pi\alpha\beta c^{-1/2}\int ds\rho\left[1+\beta\Phi+\frac{(1+\beta\Phi)^2 cc'}{\beta}\left(\frac{dt}{ds}\right)^2\right] \qquad (2.8)$$

Now if we choose

$$c = \exp(-2\beta\Phi) \qquad (2.9)$$

then (2.8) may be written

$$J\Box^2\exp(-\beta\Phi) = -8\pi\alpha\beta^2\exp(\beta\Phi)\int dt\rho c\sqrt{\left(1-\frac{v^2}{c^2}\right)}$$
$$\times\left[(1+\beta\Phi)-\frac{2(1+\beta\Phi)^2}{1-v^2/c^2}\right] \qquad (2.10)$$

where

$$\Box^2 = \frac{1}{c}\frac{\partial}{\partial t}\left(\frac{1}{c}\frac{\partial}{\partial t}\right) - \nabla^2$$

the integration variable has been changed to t. Further, if the time dependence of ρ is a delta function then

$$J\Box^2\exp(-\beta\Phi) = -8\pi\alpha\beta^2\rho(x^i, z^i(t))\sqrt{\left(1-\frac{v^2}{c^2}\right)}$$
$$\times\left[1+\beta\Phi-\frac{2(1+\beta\Phi)^2}{1-v^2/c^2}\right]\exp(-\beta\Phi) \qquad (2.11)$$

If we suppose that ρ is the mass distribution of a single particle whose coordinates of the centre of mass are Z^i and its velocity, v, is zero, then equation (2.11) becomes

$$J\Box^2\exp(-\beta\Phi) = 8\pi\alpha\beta^2\rho(x^i, z^i)(1+2\beta\Phi)(1+\beta\Phi)\exp(-\beta\Phi) \quad (2.12)$$

and for a static field we have

$$J\nabla^2 \exp(-\beta\Phi) = -8\pi\alpha\beta^2 \rho(x^i, z^i)(1 + 2\beta\Phi)(1 + \beta\Phi)\exp(-\beta\Phi) \quad (2.13)$$

We shall see in Section 4 that a comparison with the Newtonian equation of orbits requires that $2\alpha\beta^2 = -MG$. The active mass of a body is thus seen in part to derive from the field of all other masses. Equation (2.11) tells us that it is also dependent on the state of its motion. When ρ is allowed to approach a delta function, we suppose the limiting procedure to be so taken that (2.13) becomes

$$\nabla^2 \exp(-\beta\Phi) = -8\pi\alpha\beta^2 \delta^3(x^i - z^i) \quad (2.14)$$

If we wish to use (2.14) to compute the field of the earth, for example, we must bear in mind that to the lowest order the active mass of the earth is proportional to $1 + 2\beta\Phi_\odot$, where $\Phi_\odot$ is the solar potential, as can be read off from the right side of (2.13). This will be considered further in Section 4.

Aside from the terms $(1 + 2\beta\Phi)(1 + \beta\Phi)$ equation (2.13) is identical to the field equation in the second theory of Abraham (1914).

3. *Equations of Motion*

The variation

$$z^i \to z^i + \delta z^i \quad (3.1)$$

where δz^i is a function required to vanish at the end points s_1 and s_2, leads to the Euler–Lagrange equations,

$$\frac{d}{ds}(\mathcal{M}(z^i)g_{ij}(z^i)\dot{z}^j) = \frac{1}{2}\frac{\partial}{\partial z^i}(\mathcal{M}(z^i)g_{jk}(z^i))\dot{z}^j\dot{z}^k \quad (3.2)$$

where

$$\mathcal{M}(z^i) = \int_\Omega d^4x\rho(x^i, z^i)(1 + \beta\Phi(x^i))^2$$

By defining

$$m_{ij} = \mathcal{M}(z^i)g_{ij} \quad (3.3a)$$

$$m^{ij} = \frac{g^{ij}}{\mathcal{M}(z^i)} \quad (3.3b)$$

$$\begin{aligned}\Gamma^i_{jk} &= \tfrac{1}{2}m^{ir}(\partial_k m_{rj} + \partial_j m_{rk} - \partial_r m_{jk}) \\ &= \begin{Bmatrix} i \\ jk \end{Bmatrix} + \frac{1}{2\mathcal{M}}(\delta_j{}^i\partial_k\mathcal{M} + \delta_k{}^i\partial_j\mathcal{M} - g_{jk}g^{ir}\partial_r\mathcal{M})\end{aligned} \quad (3.3c)$$

where

$$\begin{Bmatrix} i \\ jk \end{Bmatrix} = \tfrac{1}{2}g^{ir}(\partial_k g_{rj} + \partial_j g_{rk} - \partial_r g_{jk}) \quad (3.3d)$$

the equations of motion (3.2) may be written

$$\ddot{z}^i + \Gamma^i_{jk}\dot{z}^j\dot{z}^k = 0 \tag{3.4a}$$

It is tempting to interpret (3.4) as an equation for geodesics. It is true that it is a geodesic equation for a manifold with the fundamental form given by

$$ds^2 = m_{ij}\,dx^i\,dx^j$$

however, the metric tensor by which indices are lowered and raised is in fact g_{ij}. Introducing the expression for Γ^i_{jk} from (3.3c), (3.4) becomes

$$\ddot{z}^i + \left\{ {i \atop jk} \right\}\dot{z}^j\dot{z}^k = (\dot{z}^i\dot{z}^k - \tfrac{1}{2}g^{ik})\,\partial_k \ln \mathscr{M} \tag{3.4b}$$

We see from (3.4b) that the motion of a particle is such that it proceeds along a geodesic path unless acted upon by a field, i.e. that the gradient of $\mathscr{M}$ differs from zero. This generalisation of Newton's law of motion also requires this motion to be rectilinear since a vanishing of the gradient implies that g_{ij} is independent of Φ (or in any event $\Phi = \text{constant}$) so that the space becomes pseudo-Euclidean and thus the motion of material particles must be along straight lines, the geodesics of this geometry.

We should say something regarding the status of the equivalence principle in the strong and weak forms in this theory. By the strong principle one usually means the covariancy of the equations. The weak principle means that the equations of motion are independent of mass. We see from (3.2) that the equations of motion will be independent of some dimensional factor called mass, but not the structure of matter. This would mean a violation, in principle, of the Eötvös experiments. However, for the objects used in these experiments, Φ is essentially a constant, as regards the definition of $\mathscr{M}$, so that for such objects the weak principle will be satisfied. The general covariancy of the equations guarantees that the strong principle is satisfied. We must thus conclude that the weak form of the equivalence principle is logically disjointed from the strong form of the equivalence principle, since this theory satisfies the latter but not the former.

4. *The One-Body Problem*

It is the purpose of this section to evaluate the constants of this theory by comparing with the Newtonian orbits. We make the following simplifying assumptions. The attraction centre is fixed in space and is describable by a delta function. The body whose motion we wish to compute is also describable by a delta function. We also take the attraction centre to be spherically symmetric. With these assumptions the field equation (2.10) reduces to

$$\frac{d}{dr}\left(r^2\frac{d\exp(-\beta\Phi)}{dr}\right) = -8\pi\alpha\beta^2\,\delta(r) \tag{4.1}$$

The solution to (4.1) satisfying the proper boundary condition at infinity is

$$\exp(-\beta\Phi) = 1 + \frac{2\alpha\beta^2}{r} \tag{4.2}$$

The gradient of the field is

$$\beta\frac{d\Phi}{dr} = \frac{2\alpha\beta^2}{r}\exp(\beta\Phi) \tag{4.3}$$

If the terms quadratic in the field are neglected we find from (4.2) that

$$\beta\Phi \approx -\frac{2\alpha\beta^2}{r} \tag{4.4}$$

so that (2.8) to the first order reads

$$c = 1 + \frac{4\alpha\beta^2}{r} \tag{4.5}$$

The equations of motion are comprised of two conservation laws and an orbit equation. We consider the motion to be planar. The conservation of energy follows from the first equation of (3.1) viz.,

$$\frac{d}{ds}\left[(1+\beta\Phi)^2 c^2 \frac{dt}{ds}\right] = 0 \tag{4.6}$$

which when integrated may be written

$$\frac{dt}{ds} = \frac{E}{(1+\beta\Phi)^2 c^2} \tag{4.7}$$

or

$$E = (1+\beta\Phi)^2 c^2 \frac{dt}{ds} \approx c\frac{dt}{ds}$$

Now using the definition of the line element then to this order we have

$$E = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \approx \frac{1}{c^2}(c^2 + \tfrac{1}{2}v^2) \tag{4.8}$$

The conservation or orbital angular momentum follows from the equation

$$-\frac{d}{ds}\left[2(1+\beta\Phi)^2 r^2 \frac{d\phi}{ds}\right] = 0 \tag{4.9}$$

which when integrated gives

$$\frac{d\phi}{ds} = \frac{h}{(1+\beta\Phi)^2 r^2} \tag{4.10}$$

The radial equation is

$$-\frac{d}{ds}\left[2(1+\beta\Phi)^2\frac{dr}{ds}\right]=2(1+\beta\Phi)\beta\frac{d\Phi}{dr}+(1+\beta\Phi)^2 \times\left[2cc'\frac{d\Phi}{dr}\left(\frac{dt}{ds}\right)^2-2r\left(\frac{d\Phi}{ds}\right)^2\right] \tag{4.11}$$

By changing the independent variable to ϕ through the

$$\frac{dr}{ds}=\frac{dr}{d\phi}\frac{d\phi}{ds}=\frac{h}{(1+\beta\Phi)^2 r^2}\frac{dr}{d\phi} \tag{4.12}$$

and substituting from (4.7) and (4.10) into (4.11) we obtain

$$\left(\frac{1}{r}\right)''+\frac{1}{r}=\left[1+\beta\Phi-\frac{2E^2\exp(4\beta\Phi)}{(1+\beta\Phi)^2}\right]\frac{(1+\beta\Phi)^2 r^2\beta}{h^2}\frac{d\phi}{dr} \tag{4.13}$$

which to the first order becomes, after substituting from (4.3),

$$\left(\frac{1}{r}\right)''+\frac{1}{r}=-\left[1-\left(\frac{4E^2-1}{2E^2-1}+3\right)\frac{2\alpha\beta^2}{r}\right]\frac{2\alpha\beta^2(2E^2-1)}{h^2} \tag{4.14}$$

By comparing with Newton's law of attraction we see that

$$MG=-2\alpha\beta^2(2E^2-1) \tag{4.15}$$

Thus we may choose

$$\alpha=-\tfrac{1}{2}M \tag{4.16}$$

and

$$\beta=\sqrt{G} \tag{4.17}$$

Thus (4.14) reads

$$\left(\frac{1}{r}\right)''+\frac{1}{r}=\left(1+\frac{v(E)\,MG}{r}\right)\frac{MG}{h^2} \tag{4.18}$$

where

$$v(E)=\frac{1}{2E^2-1}\left(\frac{4E^2-1}{2E^2-1}+3\right) \tag{4.19}$$

If terms of the order v^2/c^2 are neglected then $v=6$ and equation (4.18) may be written

$$\left(\frac{1}{r}\right)''+\left(1-\frac{6M^2G^2}{h^2}\right)\frac{1}{r}=\frac{MG}{h^2} \tag{4.20}$$

The solution to (4.20) is seen to be

$$\frac{1}{r}=\frac{MG}{h^2[1-(6M^2G^2/h^2)]}\left[1+\epsilon\cos\sqrt{\left(1-\frac{6M^2G^2}{h^2}\right)}(\phi-\phi_0)\right] \tag{4.21}$$

Equation (4.21) represents an ellipse in which the longitude of perihelion is seen to advance $3M^2G^2/h^2$ per revolution. This is the same value as predicted by Einstein's theory.

For an observer on the earth he will measure the speed of light as given by (2.8) which may be written

$$c(r) = 1 - \frac{2MG}{r} \tag{4.22}$$

Thus, Fermat's principle requires that

$$\delta \int \frac{\sqrt{(r'^2 + r^2)}}{c(r)} d\phi = 0 \tag{4.23}$$

which leads to the Euler–Lagrange equation

$$u'' + u = -\frac{c'(u)}{c(u)}(u'^2 + u^2) \tag{4.24}$$

where $u = 1/r$. A zeroeth order solution to this equation is

$$u = \frac{1}{R}\cos\phi \tag{4.25}$$

where R is the radius at the limb of the sun. The next higher order solution is

$$u = \frac{1}{R}\left(\cos\phi + \frac{2MG}{R}\right) + \frac{2M^2G^2}{R^3}(\cos\phi + \phi) \tag{4.26}$$

Thus, the light ray is deflected through an angle $4MG/R$. This is the same prediction as made by Einstein.

The same result could have been arrived at from different considerations. The line element for the present is

$$\lambda\, ds^2 = c^2(r)\, dt^2 - dr^2 - r^2\, d\phi^2 \tag{4.27}$$

where we postulate that $\lambda = 1$ for material particles and $\lambda = 0$ for photons. Using the two first integrals from the equations motion, (4.7) and (4.10), the line element may be written in the form

$$\lambda \frac{c^2}{h^2}(1 + \beta\Phi)^4 = \frac{E^2}{h^2} - c^2(u'^2 + u^2) \tag{4.28}$$

If we set $\lambda = 0$ and differentiate with respect to ϕ we obtain (4.24), thus a light ray is a path of null distance.

5. *Summary and Consequences*

How does one construct a massless scalar field when the metric tensor depends on the scalar field, such that a vanishing of the scalar field implies a Minkowskian metric? This question has been answered and a tenable theory of gravitation is the result.

The constants α and β were not uniquely determined but only the product $\alpha\beta^2$. We could have chosen $\beta = 1$, for example, so that α would be the gravitational radius ($MG/C_0{}^2$ in conventional units). This is apparent, as well, from the postulated form of the Lagrangian density. The coupling of this scalar field to the electromagnetic field (which will be presented in a future paper) does not further restrict these constants since it would mean simply a readjustment of the electromagnetic coupling constant.

The motion of material particles was seen to be governed by an equation which resembled in form the usual equation of geodesics of a Riemann manifold. It is, however, not a geodesic since the affine connection is not derived from the natural metric tensor. This situation cannot be remedied by introducing a new parameter for the world distance since the action principle is not invariant under general transformations but only the constant translation group. This error has been made by Thirring (1961) and Sexl (1967).

We may interpret the term $(1 + \beta\Phi)^2$ as the contribution to the inertial mass due to the presence of other bodies. The gravitational mass of a body is to the lowest order proportional to $1 - 2\beta\Phi$. This means that the earth's gravitational field should vary by 2/3 parts in 10^9 in the course of its annual motion about the sun. This means, for example, that the radius of a 30,000 km radius circular satellite orbit would vary about 2 cm. These types of effects will be considered in a future paper in regard to the earth–moon dynamics. The main result is that a light signal bounced off the moon shows a two nanosecond difference between aphelion and perihelion.

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